AEROELASTIC STABILITY OF PLANE SANDWICH-TYPE STRUCTURES PLACED IN A CURRENT OF SUPERSONIC GAS

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AEROELASTIC STABILITY OF PLANE SANDWICH-TYPE STRUCTURES PLACED IN A CURRENT OF SUPERSONIC GAS

by

L. Librescu and E. Malaiu[1]

ABSTRACT: Particular attention is devoted to the problem of the aeroelastic stability of infinite sandwich panels. The influence of the parameter which expresses the flexibility of the transverse shear of the structure and the parameter connected with the axial loads on the critical magnitudes (flutter and divergence) is investigated. The conclusions reached in other recent works regarding the lack of agreement between a series of results in this field obtained in exact and approximate fashion are examined and justified. A71-23609

A theoretical study is presented of the aeroelastic stability, in linear /171* formulation, of rectangular plane panels of the sandwich type placed in a current of ideally conducting supersonic gas.

Particular attention is devoted to the problem of stability of an infinite sandwich band. Within this context the influence is studied which is exerted by the parameters k_0^2 and R_{11} (the former expressing the transverse shear flexibility of the structure, and the latter relating to the axial loads) on the critical (flutter and divergent) values.

The conclusions stated in certain recent studies regarding the discrepancy among a number of results in this field obtained by exact and approximate methods are examined and substantiated.

1. Introduction

Review of the vast body of literature which has appeared in the field of aeroelastic stability of panels permits the conclusion that the classical theory of thin panels (a theory based on the Love-Kirchhoff hypothesis) has been given almost exclusive consideration in studies in this field. Insofar as the authors know, there are but few studies in which transverse shear deformations have been taken into account in dealing with the problem of aeroelastic stability (in this connection see [1] - [10]).

1. Institute of Fluid Mechanics

* Numbers in the margin indicate pagination in the foreign text.

As is known (see, for example, [11] - [14]), when the problems of statics and dynamics of thick (isotropic or anisotropic) panels, or those of geometrically thin panels but ones made of a material with a high degree of anisotropy, are dealt with, the effect of transverse shear deformations can no longer be ignored. Thus study of the aeroelastic stability of homogeneous plates whose transversally isotropic material has elastic constant ratios of $20 < E/G' < 50$ (E being Young's modulus in the plane of isotropy of the material and G' the shear modulus in the plane normal to the plane of isotropy, as is the case, for example, with plates made of pyrolytic graphite), and study of the aeroelastic stability of sandwich-type panels, are also cases in which the structural equations can no longer be derived by adopting the Love-Kirchhoff hypothesis as a basis. /172

The present paper is devoted to study of the aeroelastic stability of symmetrically built plane rectangular panels of the sandwich type placed in a current of supersonic gas.

2. Hypotheses. Geometric Considerations. Basic Equations.

Let a symmetrically built plane rectangular (a x b) plate of the sandwich type be placed in a current of supersonic, non-viscous, ideally conducting gas (Figure 1). We shall assume two cases of flow of the current of gas:

- case a: the current of gas flows parallel to the axis $O\bar{\xi}_1$, on the outer edge of the panel, at a velocity of U_∞^+, ($U_\infty^- = 0$), the lower surface of the panel being regarded as placed in calm air which is a poor conductor of electricity;

- case b: the current of gas flows along both surfaces of the panel, at a velocity of $U_\infty^+ = U_\infty^-$.

In both cases the magnetic field is assumed to be uniform and parallel to the undisturbed current of gas U_∞.

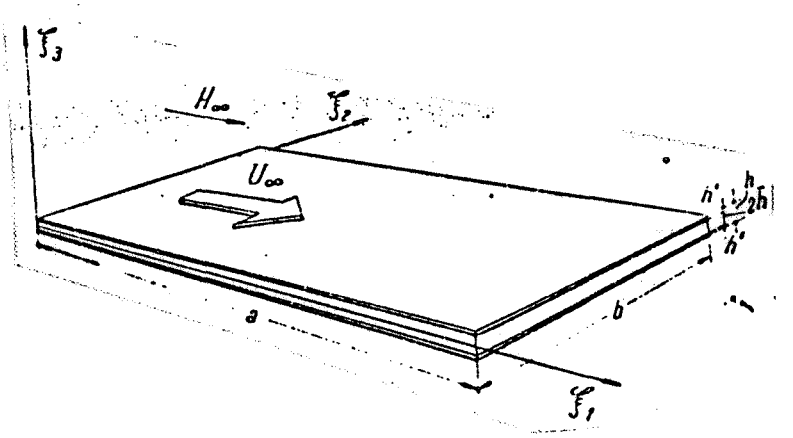

Figure 1.

The following assumptions are also made: in case a, $H_\infty = H_\infty^+$ ($H_\infty^- = 0$); in case b, $H_\infty^+ = H_\infty^-$, where H_∞ represents the component of the magnetic field parallel to the $O\xi_1$ axis.

We shall assume zero electromagnetic effects in the material of the plate.

In case a the magnetoaerodynamic operator is expressed by [15]:

$$\mathcal{A}(w) = -\frac{\rho_\infty U_\infty (m^2-1)}{m^2(K^2-1)^{1/2}}\left(\frac{\partial w}{\partial t} + U_\infty \frac{\partial w}{\partial \xi_1}\right) \qquad (1)^*$$

and in case b by: /173

$$\mathcal{A}(w) = -\frac{2\rho_\infty U_\infty (m^2-1)}{m^2(K^2-1)^{1/2}}\left(\frac{\partial w}{\partial t} + U_\infty \frac{\partial w}{\partial \xi_1}\right), \qquad (2)$$

where m is the Alfven number, defined by $m = U_\infty/\alpha$, that is, the ratio of velocity U_∞ of the undisturbed current to the velocity of propagation of Alfven waves; and:

$$\alpha = \frac{H_\infty}{(4\pi\rho_\infty)^{1/2}}; \qquad K^2 = \frac{(M_\infty^2-1)(m^2-1)}{M_\infty^2+m^2-1} + 1. \qquad (3)$$

It is to be seen that in the absence of the magnetic field ($H_\infty \to 0$) the Alfven number $m \to \infty$; $K^2 \to M^2 - 1$, and thus (1) and (2) pass, respectively, to the well-known Ackeret formulas:

$$\mathcal{A}(w) = \frac{\rho_\infty U_\infty}{\sqrt{M_\infty^2-1}}\left(\frac{\partial w}{\partial t} + U_\infty \frac{\partial w}{\partial \xi_1}\right), \qquad (4)$$

$$\mathcal{A}(w) = -\frac{2\rho_\infty U_\infty}{\sqrt{M_\infty^2-1}}\left(\frac{\partial w}{\partial t} + U_\infty \frac{\partial w}{\partial \xi_1}\right).$$

* The expression for magnetoaerodynamic pressure given by relation (1) was used in studies of the stability of plane panels by Lisunov [16] and Efremov [9].

As regards the resolvent equations of the theory of bending of symmetrically built elastic plane structures of the sandwich type, they have been derived in a general context in [17].

In presenting the expression for the structural operator we shall confine ourselves to the case of the linear theory of symmetrically built rectangular sandwich plates (geometric and physical-mechanical symmetry) of two thin, isotropic, outer layers separated by a thick median layer consisting of a transversally isotropic material (the plane of isotropy of the core material coinciding with the plane $\xi_3 = 0$). We shall additionally assume that the boundary equation, corresponding to transverse shear, may be ignored in the problem of vibrations and stability of sandwich structures (in this connection see [18] and [19]). Under these conditions the transverse bending equation in the theory of symmetrically built sandwich plates is given by

$$\mathcal{D}\Delta\Delta w + T_{11}^0 w_{,11} + T_{22}^0 w_{,22} - k^2[T_{11}^0(w_{,1111} + w_{,1122}) + T_{22}^0(w_{,2211} + w_{,2222})] - q + k^2(q_{,11} + q_{,22}) = 0, \tag{5}$$

where $\mathcal{D}$ represents the rigidity in transverse bending, /174

$$\mathcal{D} = \frac{2}{3}\left[\frac{\bar{E}\bar{h}^3}{1-\bar{\nu}^2} + \frac{E'<(\bar{h}+h')^3 - \bar{h}^3>}{1-\nu'^2}\right], \tag{6}$$

$$k^2 = \frac{1}{\bar{G}_3}\left[\frac{2}{3}\,\frac{\bar{G}\bar{h}^2}{1-\bar{\nu}} + \frac{2G'\bar{h}h'}{1-\nu'}\right]$$

(in the classic panel theory $k^2 \to 0$ ($\bar{G} \to \infty$)); T_{11}^0, T_{22}^0 are the normal loads of the plane of the plate (positive in compression);

$$\bar{G} = \frac{\bar{E}}{2(1+\bar{\nu})}; \qquad G' = \frac{E'}{2(1+\nu')},$$

$\bar{E}$, $\bar{\nu}$; E', ν' are respectively Young's modulus and the Poisson constant corresponding to the median layer and the upper layer, respectively;

$\bar{G}_3$ is the shear modulus corresponding to the core material in the plane transverse to the median plane;

$2\bar{h}$; $h' = h''$ are the thicknesses respectively of the median layer and of the outer layer;

$h = 2(\bar{h} + h')$ is the total thickness of the sandwich plate;

q is the load normal to the median plane of the plate, expressed by

$$q = -m_0\ddot{w} - m_0\varepsilon\dot{w} + \delta p,$$

where m_0 is the reduced mass of the structure and ε the structural damping factor; $\delta p = \mathcal{A}(w)$;

Δ is the two dimensional Laplacian.

In all places in this study, quantities appearing with a bar or an accent identify properties of the median layer and of the upper layer, respectively, while the partial derivative with respect to coordinate ξ_i $(i = 1,2,3)$ will be indicated by subscript i preceded by a comma $(\frac{\partial}{\partial \xi_i}(\ ) = (\ )_{,i})$; a point above a quantity denotes partial derivation with respect to time t.

Under these conditions the aeroelastic equilibrium equation is expressed by

$$\mathcal{D}\Delta\Delta w + T_{11}^0 w_{,11} + T_{22}^0 w_{,22} - k^2[T_{11}^0(w_{,1111} + w_{,1122}) + T_{22}^0(w_{,2211} + w_{,2222})] + \tag{7}$$

$$+ m_0\ddot{w} + m_0\varepsilon\dot{w} - k^2 m_0\ddot{w}_{,11} - k^2 m_0\ddot{w}_{,22} + \frac{\rho_\infty U_\infty^2(m^2-1)}{m^2(K^2-1)^{1/2}} w_{,1} -$$

$$- k^2\frac{\rho_\infty U_\infty^2(m^2-1)}{m^2(K^2-1)^{1/2}} w_{,111} - k^2\frac{\rho_\infty U_\infty^2(m^2-1)}{m^2(K^2-1)^{1/2}} w_{,122} + \frac{\rho_\infty U_\infty(m^2-1)}{m^2(K^2-1)^{1/2}}\dot{w} = 0,$$

which can be more concisely written as 175

$$(\mathcal{S} - \mathcal{A} - \mathcal{J} + k^2\Delta\mathcal{A} + k^2\Delta\mathcal{J})(w) = 0, \tag{8}$$

where $\delta(w)$ represents the structural operator and $\mathcal{A}(w)$ and $\mathcal{J}(w)$ respectively the aerodynamic and the inertial operator.

3. Another Form for the Aeroelastic Equilibrium Equation.

In what follows we shall undertake to convert resolvent equation (7) to a form more favorable for analysis of aeroelastic stability (see [19], [21]). To this end we shall represent $w(\xi_1, \xi_2, t)$ by

$$w(\xi_1, \xi_2, t) = \sum_{m,n=1}^{\infty} w_{mn}(t) f_{mn}(\xi_1, \xi_2), \tag{9}$$

where the functions $f_{mn}(\xi_1, \xi_2)$ satisfy all the boundary conditions as regards normal displacement, $w_{mn}(t)$ being the time functions to be determined.*

It is a noteworthy fact that, in contrast to the case in which the effect of transverse shear deformations is disregarded, in the present case expressed

* Expressions for $f_{mn}(\xi_1, \xi_2)$ in several boundary condition cases are given in [20] and [21, p. 74].

in terms of aerodynamics (more precisely in terms of $k^2 \Delta \mathcal{A}(w)$) coupling of the modes according in ξ_2 occurs as well.

In study of the aeroelastic stability of plane structures of the sandwich type this requires that $w(\xi_1, \xi_2, t)$ be represented in conformity with equation (9); the same representation of $w(\xi_1, \xi_2, t)$ is obviously also required in study of the aeroelastic stability of homogeneous plates whenever the effect of transverse shear deformations is taken into account.

Hence, the representation of $w(\xi_1, \xi_2, t)$ by

$$w = \sum_{m=1}^{\infty} w_{mn}(t) f_{mn}(\xi_1, \xi_2), \tag{9'}$$

such as is encountered in a number of recent studies, appears to be rather debatable in such cases.

By applying the Galerkin technique to equation (8):

$$\int_0^a \int_0^b (\mathcal{S} - \mathcal{A} - \mathcal{J} + k^2 \Delta \mathcal{J} + k^2 \Delta \mathcal{A})(w) f_{rn} \, d\xi_1 \, d\xi_2 = 0 \tag{10}$$

$$(r = 1,2,\ldots; \quad n = 1,2,\ldots),$$

one obtains the following system of linear equations, a system linear with respect to the coefficients of the series: /176

$$\ddot{w}_{rn} + \varepsilon_r \dot{w}_{rn} + \Omega_{rn}^2 w_{rn} + F_{rn} = 0, \tag{11}$$

which lends itself better to analysis of aeroelastic stability than does the equation with partial derivatives (7).

In order not to be led into exorbitantly complicated expressions we shall confine the analysis to the case of aeroelastic stability of an infinite sandwich band (Figure 2).

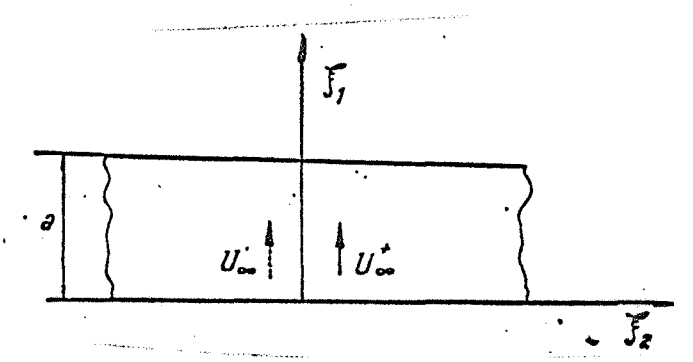

Figure 2.

In this case we shall assume $w = w(\xi_1, t)$ and shall consider two support condition cases:

- case A: the edges perpendicular to the direction of flow of the gas current are simply supported.

- case B: the edges perpendicular to the direction of flow of the gas current rigidly fixed.

The expressions for $w(\xi_1, t)$ in the two support cases in question are given respectively by

$$w(\xi_1, t) = \sum_{r=1}^{\infty} w_r(t) \sin \frac{r\pi\xi_1}{a},$$
$$w(\xi_1, t) = \sum_{r=1}^{\infty} w_r(t) \left(\cos \frac{(r-1)\pi\xi_1}{a} - \cos \frac{(r+1)\pi\xi_1}{a} \right). \quad (12)$$

If the Galerkin technique is employed, in the case of an infinite band there are obtained the equations

$$\frac{d^2\zeta_r}{d\tau^2} + \varepsilon_r \frac{d\zeta_r}{d\tau} + w_r^2 \zeta_r + F_r(\zeta_p, M) = 0 \qquad (r = 1,2,\ldots), \quad (13)$$

in which the dimensionless variables

$$\tau = \Omega t; \qquad \zeta_r = \frac{w_r}{h}, \quad (14)$$

have been introduced, as have the dimensionless parameters /177

$$R_{11} = \frac{T_{11}^0 a^2}{\pi^2 D}; \qquad \nu = \frac{h M_\infty^2 (m^2 - 1)}{a m^2 (K^2 - 1)^{1/2}}, \quad (15)$$

$$\Omega^2 = \frac{\pi^4 D}{m_0 a^4}; \qquad P_1 = \frac{\varkappa p_\infty}{m_0 h \Omega^2}.$$

In the case of flow on a single surface of the panel we have

$$\varepsilon_r = \frac{1}{\Omega} \left(\varepsilon + \frac{\varkappa p_\infty}{m_0 c_\infty} \right); \qquad P = P_1, \quad (16)$$

and in the case of flow on both surfaces of the panel at a velocity of $U_\infty^+ = U_\infty^- = U_\infty$, we have

$$\varepsilon_r = \frac{1}{\Omega} \left(\varepsilon + \frac{2\varkappa p_\infty}{m_0 c_\infty} \right); \qquad P = 2P_1. \quad (17)$$

The other expressions for the parameters are identical in the case of flow on one or both surfaces of the panel; in support case A

$$\omega_r^2 = \frac{r^4}{1 + k_0^2 r^2} - R_{11} r^2,$$

the natural vibrational frequencies of the structure in vacuo being

$$F_r = -4\lambda \frac{r}{1 + k_0^2 r^2} \sum_{p=1}^{\infty} \frac{2p + 1 - r}{(2p+1)(2p+1-2r)} \{1 + k_0^2 (2p+1-r)^2\} \zeta_{2p+1-r} \quad (18)$$
$$(r = 1,2,\ldots),$$

and in support of case B

$$\omega_1^2 = [16 - 4R_{11}(1 + 4k_0^2)](3 + 4k_0^2)^{-1}, \tag{19}$$

$$\omega_r^2 = [r^4 + 6r^2 + 1 - R_{11} < r^2 + 1 + k_0^2(r^4 + 6r^2 + 1) >] [1 + k_0^2(r^2 + 1)]^{-1}$$

$$(r = 2, 3, \ldots),$$

$$F_1 = -\frac{128}{15}\lambda(1 + 4k_0^2)(3 + 4k_0^2)^{-1}\zeta_2,$$

$$F_r = -2\lambda(1 + k_0^2(r^2 + 1))^{-1} \times$$

$$\times \sum_{t=1}^{\infty} \left\{ \frac{(2t - r)^2}{(2t - 1)(2t + 1 - 2r)} (1 + k_0^2(2t - r)^2) - \right.$$

$$- \frac{(2t - r)^2}{(2t + 1)(2t - 1 - 2r)} (1 + k_0^2(2t - r)^2) -$$

$$- \frac{(2t + 2 - r)^2}{(2t + 1)(2t + 3 - 2r)} (1 + k_0^2(2t + 2 - r)^2) +$$

$$\left. + \frac{(2t + 2 - r)^2}{(2t + 3)(2t + 1 - 2r)} (1 + k_0^2(2t + 2 - r)^2) \right\} \zeta_{2t+1-r}$$

$$(r = 2, 3, \ldots),$$

where $k_0^2 = k^2\pi^2/a^2$; $\lambda = P\nu$ are, respectively, the parameter characterizing the transverse shear rigidity and the new velocity parameter. /178

If the successive modes ζ_p; ζ_{p+1} are considered, there are obtained in support case A the equations

$$\frac{d^2\zeta_p}{d\tau^2} + \varepsilon_r \frac{d\zeta_p}{d\tau} + \omega_p^2\zeta_p - \frac{4p(1 + p)}{1 + 2p}\lambda\rho^2\zeta_{p+1} = 0, \tag{20}$$

$$\frac{d^2\zeta_{p+1}}{d\tau^2} + \varepsilon_r \frac{d\zeta_{p+1}}{d\tau} + \omega_{p+1}^2\zeta_{p+1} + \frac{4p(1 + p)}{1 + 2p} \frac{\lambda}{\rho^2}\zeta_p = 0, \tag{21}$$

where

$$\rho^2 = \frac{1 + k_0^2(p + 1)^2}{1 + k_0^2p^2} \qquad (p = 1, 2, \ldots),$$

and in support case B

$$\frac{d^2\zeta_1}{d\tau^2} + \varepsilon_r \frac{d\zeta_1}{d\tau} + 4[4 - R_{11}(1 + 4k_0^2)](3 + 4k_0^2)^{-1}\zeta_1 - \tag{22}$$

$$- \frac{128}{15}\lambda(1 + 4k_0^2)(3 + 4k_0^2)^{-1}\zeta_2 = 0,$$

$$\frac{d^2\zeta_2}{d\tau^2} + \varepsilon_r \frac{d\zeta}{d\tau} + [41 - R_{11}(5 + 41k_0^2)](1 + 5k_0^2)^{-1}\zeta_2 +$$

$$+ \frac{64}{15}\lambda(1 + 5k_0^2)^{-1}(1 + 4k_0^2)\zeta_1 = 0$$

and

$$\frac{d^2\zeta_p}{d\tau^2} + \varepsilon_r \frac{d\zeta_p}{d\tau} + [p^4 + 6p^2 + 1 - R_{11}(p^4 + 6p^2 + 1)](1 + k_0^2(p^2 + 1))^{-1}\zeta_p -$$

$$- 2\lambda(1 + k_0^2(p^2 + 1))^{-1}\left\{\frac{p^2(1 + k_0^2 p^2)}{2p - 1} + \frac{p^2(1 + k_0^2 p^2)}{2p + 1} - \right.$$

$$\left. - \frac{(p + 2)^2(1 + k_0^2(p + 2)^2)}{3(2p + 1)} + \frac{(p + 2)^2(1 + k_0^2(p + 2)^2)}{2p + 3}\right\}\zeta_{p+1} = 0,$$

$$\frac{d^2\zeta_{p+1}}{d\tau^2} + \varepsilon_r \frac{d\zeta_{p+1}}{d\tau} + [1 + k_0^2((p + 1)^2 + 1)]^{-1} \times [(p + 1)^4 + 6(p + 1)^2 +$$

$$+ 1 - R_{11}[(p + 1)^2 + 1 + k_0^2 < (p + 1)^4 + 6(p + 1)^2 + 1 >]]\zeta_{p+1} -$$

$$- 2\lambda(1 + k_0^2 < (p + 1)^2 + 1 >)\left\{\frac{(p + 1)^2(1 + k_0^2(p - 1))^2}{(1 - 2p)} + \right.$$

$$+ \frac{(p - 1)^2(1 + k_0^2(p - 1)^2)}{3(2p + 1)} - \frac{(p + 1)^2(1 + k_0^2(p + 1)^2)}{2p + 1} -$$

$$\left. - \frac{(p + 1)^2(1 + k_0^2(p + 1)^2)}{(2p + 3)}\right\}\zeta_p = 0 \qquad (p = 2,3,\ldots).$$

Equations (21) and (22) will be used in study of the aeroelastic stability of an infinite sandwich band in this last form. /179

4. Analysis of the Aeroelastic Stability of an Infinite Sandwich Band.

The problem of aeroelastic stability of an infinite sandwich band will be studied on the basis of equations (21) and (22), obtained respectively in case A and B.

If solutions of the form $\zeta_m = C_m e^{wt}$ are considered, where $C_m (m = p, p + 1)$ are arbitrary constants, and if the condition is set of having nontrivial solutions, there is obtained the characteristic equation

$$(\omega_p^2 - \zeta)(\omega_{p+1}^2 - \zeta) + b_p b_{p+1}\lambda^2 = 0, \qquad (23)$$

where the frequency parameter ζ is defined by

$$-\zeta = \left(\frac{\omega}{\Omega}\right)^2 + \varepsilon_r \frac{\omega}{\Omega}. \qquad (24)$$

For a given value of ζ two values are obtained from (24) for frequency w:

$$\omega_{(1),(2)} = -\frac{\varepsilon_T \Omega}{2} \pm \left[\left(\frac{\varepsilon_T \Omega}{2}\right)^2 - \Omega^2 \zeta\right]^{1/2}. \tag{25}$$

Since w is in the general case a complex quantity ($w = \mathrm{Re}\, w + i\,\mathrm{Im} w$; $i = (-1)^{1/2}$), it is to be inferred from (24) that one of the two roots, say $w_{(1)}$ possesses the property*

$$\mathrm{Re}\, w_{(1)} < 0.$$

The second frequency $w_{(2)}$ satisfies the conditions $\mathrm{Re} w_{(2)} < 0$; $\mathrm{Re} w_{(2)} = 0$; or <u>/180</u> $\mathrm{Re} w_{(2)} > 0$ if, and only if,

$$\zeta = \mathrm{Re}\zeta(\lambda) + i\mathrm{Im}\zeta(\lambda) \tag{26}$$

is inside, on the boundary, or outside the "stability parabola," which is defined by**

$$(\mathrm{Im}\,\zeta(\lambda))^2 = \varepsilon_T^2 \mathrm{Re}\,\zeta(\lambda), \tag{26'}$$

where ε_T was considered to be the same for all the modes.

(26) and (23) being taken into account and the real and the imaginary portions being separated, two polynomials are obtained:

$$\begin{aligned} &\mathrm{Re}: \ \omega_p^2\omega_{p+1}^2 - \zeta_R(\omega_p^2 + \omega_{p+1}^2) + (\zeta_R^2 - \zeta_I^2) + (b_p b_{p+1}\lambda)^2 = 0, \\ &\mathrm{Im}: \ 2\zeta_R - (\omega_p^2 + \omega_{p+1}^2) = 0, \end{aligned} \tag{27}$$

where

$$\zeta_R \equiv \mathrm{Re}\zeta; \qquad \zeta_I \equiv \mathrm{Im}\zeta.$$

If the condition is set that the system be on the threshold of instability of the flutter type (a condition which is realized when (26) and (27) are taken into account), and if it is required that system (27) have a single solution (a condition satisfied if, and only if, the Sylvester determinant corresponding to the system of equation (27) (in which (26') is first taken into account) is zero), the expression for the critical flutter velocity is obtained:

$$\lambda_*^2 = \frac{1}{4b_p b_{p+1}} [(\omega_{p+1}^2 - \omega_p^2)^2 + 2\varepsilon_T^2(\omega_{p+1}^2 + \omega_p^2)], \tag{28}$$

---

* In keeping with the solutions under consideration the initial state of equilibrium is stable, neutrally stable, or unstable (flutter-type instability) if, and only if, $\mathrm{Re}\, w < 0$, $\mathrm{Re}\, w = 0$, or $\mathrm{Re}\, w > 0$.

** The concept of "stability parabola" was introduced in the study of aeroelastic stability by Movčian [22], [23] (also see Bolotin [24]) and subsequently used in the same field by Stepanov [25], Krumhaar [26], and Grigoliuk and Mihailov [3]. In the broader context of general non-conservative stability the concept of "stability parabola" was introduced by Leipholz [27], [28].

as is also the critical flutter frequency:

$$\zeta_R = \frac{1}{2}(\omega_{p+1}^2 + \omega_p^2). \tag{29}$$

Considering the first two vibration modes (p = 1), in support case A we have

$$b_1 = b_2 = \frac{8}{3}, \tag{30}$$

and in support case B /181

$$b_1 = \frac{128}{15}\frac{1+4k_0^2}{3+4k_0^2}; \quad b_2 = \frac{64}{15}\frac{1+4k_0^2}{1+5k_0^2}, \tag{31}$$

w_1^2 and w_2^2 being the natural vibrational frequencies, which in the two support cases have been defined respectively by $(19)_1$ and $(19)_2$.

Let us examine the case corresponding to negligibly small ε_T, where we shall consider solutions of the form $\zeta_m = C_m e^{iwt}$. Under these conditions, from equations (22) (affected by the circumstance that ε_T is negligibly small) we obtain in support case A

$$\begin{aligned} &(\omega_p^2 - \omega^2) C_p - \frac{4p(1+p)}{1+2p}\lambda\rho^2 C_{p+1} = 0, \\ &\frac{4p(1+p)}{1+2p}\frac{\lambda}{\rho^2} C_p + (\omega_{p+1}^2 - \omega^2) C_{p+1} = 0, \end{aligned} \tag{32}$$

C_p being arbitrary constants (p = 1,2,...), and in support case B, the first two successive modes ζ_1, ζ_2 (p = 1) and respectively the modes ζ_1, ζ_2 (p = 2,3...) being taken into account, we obtain

$$\begin{aligned} &(\omega_1^2 - \omega^2) C_1 - \frac{128}{15}\lambda\frac{1+4k_0^2}{3+4k_0^2} C_2 = 0, \\ &\frac{64}{15}\lambda\frac{1+4k_0^2}{1+5k_0^2} C_1 + (\omega_2^2 - \omega^2) C_2 = 0 \end{aligned} \tag{33}$$

and accordingly

$$\begin{aligned} &(\omega_p^2 - \omega^2) C_p - \lambda b_{p+1} C_{p+1} = 0, \\ &\lambda b_p C_p + (\omega_{p+1}^2 - \omega^2) C_{p+1} = 0 \quad (p = 2, 3, \ldots). \end{aligned} \tag{34}$$

Setting the condition of nontriviality of constants C_p, we obtain the characteristics equation written in the general form

$$\lambda^2 b_p b_{p+1} = (\omega_p^2 - \omega^2)(\omega^2 - \omega_{p+1}^2), \tag{35}$$

which can be rendered special for support cases A and B, respectively. For $\lambda = 0$ the frequencies obtained correspond to the natural vibrational frequencies w_p^2, w_{p+1}^2.

With increase in velocity parameter λ, for a certain value of the latter $\lambda = \lambda_{*fl}$ the characteristic equation has a double nonzero frequency (for $\lambda > \lambda_{*fl}$ the characteristic equation has a pair of complex conjugate roots). The state of equilibrium as thus defined is an unstable state of the flutter type for all velocities $\lambda \geq \lambda_{*fl}$, and λ_{*fl} represents the critical velocity of loss of stability of the flutter type (defined for the case of negligibly small ε_T).

Setting the condition of fusion of frequencies $w_{(1)}^2$ and $w_{(2)}^2$, we obtain the critical flutter velocity and frequency, defined by

$$\lambda_{*fl} = \frac{1}{4b_p b_{p+1}} (\omega_{p+1}^2 - \omega_p^2)^2, \tag{36}$$

$$\omega_*^2 = \frac{1}{2} (\omega_{p+1}^2 + \omega_p^2).$$

In keeping with $w = 0$, from (35) we determine the equation for loss of stability through divergence:

$$\lambda^2 b_p b_{p+1} + \omega_p^2 \omega_{p+1}^2 = 0. \tag{37}$$

Relations (35), (36) and (37), which can be rendered special for the two boundary condition cases A and B, represent the foundation for study of the aeroelastic stability of an infinite sandwich band. In what follows we shall express the dependence of the critical quantities as a function of parameter k_0^2 (parameter k_0^2 as defined by (20) corresponds to the theory of bending of a sandwich plate with a rigid core; in the case in which $\bar{E} \to 0$ is assumed in (2), the expression is obtained for k_0^2 corresponding to the theory of sandwich plates with a light core; if $h' \to 0$ is assumed in (20), the expression is obtained for this parameter proper to the theory of bending of homogeneous plates, the effect of transverse shear deformations being taken into account).

Lastly, in order to clarify the influence of the Alfven number on the critical flutter velocity, from (20), $(15)_2$, and (3) the relation

$$h_* = \frac{M_\infty^2}{\lambda_* (M_\infty^2 - 1)^{1/2}} \left(1 + \frac{M_\infty^2 - 2}{m^2} + \frac{1 - M_\infty^2}{m^4}\right)^{1/2}, \tag{38}$$

is obtained, and from the latter the influence of the Alfven number on the critical flutter velocity is determined.

5. Numerical Example

In this section we shall clarify, on the basis of numerical results, the influence of parameters k_0^2 and R_{11} on the critical quantities (flutter and divergence).

For this purpose we shall give the most important relations corresponding to support cases A and B. The relations presented are obtained by particularizing the general relations derived in Section 4, by taking the first two vibrational modes into account and assuming the damping factor ε_T to be negligibly small.

Thus, in case A we have /183

$$\lambda = \frac{3}{8}\left[(\omega^2 - \omega_1^2)\,(\omega_2^2 - \omega^2)\right]^{1/2}, \tag{39}$$

$$\lambda_{*\mathrm{div}} = \frac{3}{8}\left[\left(R_{11} - \frac{1}{1+k_0^2}\right)\left(\frac{16}{1+4k_0^2} - 4R_{11}\right)\right]^{1/2} \tag{40}$$

From the condition $\frac{\partial \lambda_{*\mathrm{div}}}{\partial R_{11}}$ there are derived, with the coordinates $(\bar{\lambda},\ \bar{R}_{11})$ of the boundary point:

$$\bar{\lambda} = \frac{9}{8(1+k_0^2)\,(1+4k_0^2)},$$

$$\bar{R}_{11} = \frac{5+4k_0^2}{2(1+k_0^2)\,(1+4k_0^2)}, \tag{41}$$

$$\lambda_{*fl} = \frac{3}{16}\left[\frac{16}{1+4k_0^2} - \frac{1}{1+k_0^2} - 3R_{11}\right],$$

$$\omega_1^2 = \frac{1}{1+k_0^2} - R_{11};\quad \omega_2^2 = \frac{16}{1+4k_0^2} - 4R_{11}, \tag{42}$$

and in case B

$$\lambda = \frac{15}{64\times 2^{1/2}}\,\frac{[(3+4k_0^2)\,(1+5k_0^2)]}{(1+4k_0^2)}\,\left[(\omega_1^2 - \omega^2)\,(\omega^2 - \omega_2^2)\right]^{1/2}, \tag{43}$$

$$\lambda_{*\mathrm{div}} = \frac{15}{32\times 2^{1/2}}\,\frac{1}{1+4k_0^2}\,[[(1+4k_0^2)\,R_{11} - 4]\,[41 - R_{11}(5+41k_0^2)]]^{1/2},$$

with boundary-point coordinates ($\bar{\lambda}$, $\bar{R}_{11}$):

$$\bar{\lambda} = \frac{15}{32 \times 2^{1/2}} \frac{1}{1 + 4k_0^2} \left[\left(\frac{61 + 328\,k_0^2}{2(5 + 41\,k_0^2)} - 4 \right) \left(41 - \frac{61 + 328\,k_0^2}{2(1 + 4k_0^2)} \right) \right]^{1/2},$$

$$\bar{R}_{11} = \frac{61 + 328\,k_0^2}{2(1 + 4k_0^2)\,(5 + 41k_0^2)},$$

$$\lambda_{*fl} = \frac{15}{128 \times 2^{1/2}} \frac{[(3 + 4k_0^2)\,(1 + 5k_0^2)]^{1/2}}{1 + 4k_0^2} (\omega_2^2 - \omega_1^2),$$

with

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$$\omega_1^2 = \frac{4[4 - R_{11}(1 + 4\,k_0^2)]}{3 + 4k_0^2},$$

$$\omega_2^2 = \frac{41 - R_{11}\,(5 + 41k_0^2)}{1 + 5k_0^2}.$$

The relations $\lambda_{*fl} \sim k_0^2$ (Figure 3) and $\lambda_{*fl} \simeq R_{11}$ (Figure 4) are derived on the basis of relations (41) and (45) for cases A and B respectively.

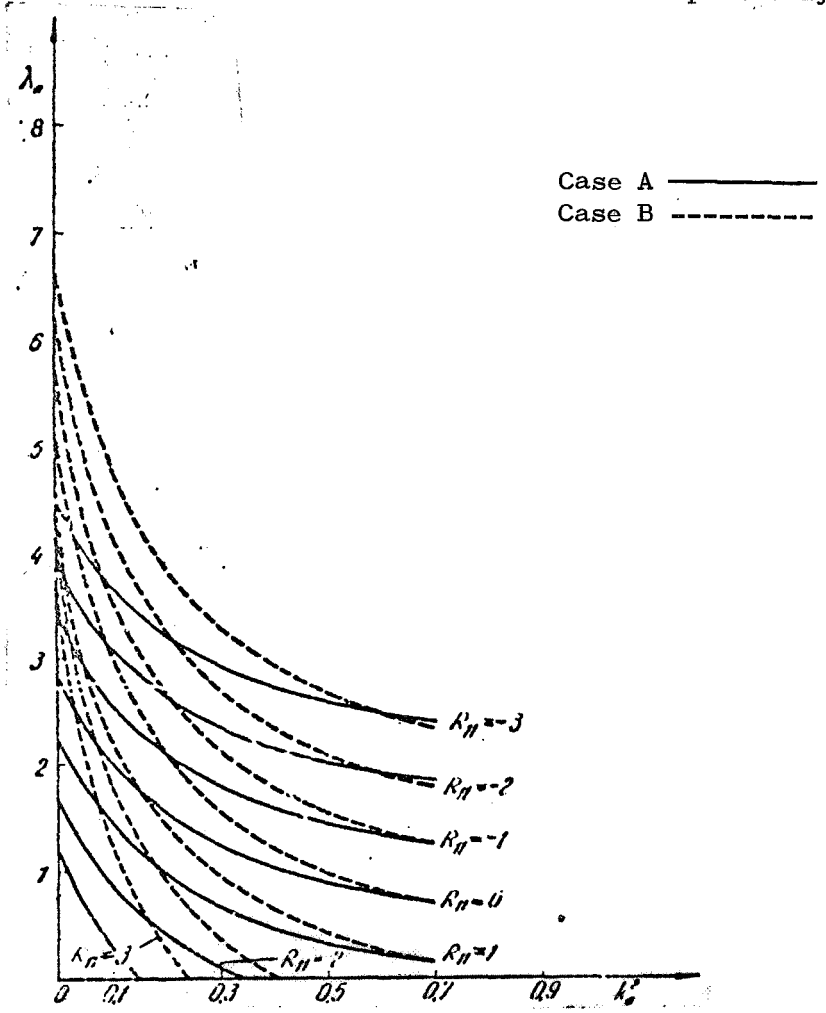

Figure 3.

The stability region of the plane form of equilibrium of a sandwich structure is shown in Figure 5b, in the plane of coordinates (λ, R_{11}). This region, bounded by these coordinates, segment BC of curve CBD, and line AB (see Figure 5a), represents in the plane of the parameters in question equations /185 (40), (41) and (44), (45) corresponding respectively to case A and case B. Curve BC corresponds to the boundary of static loss of stability (due to divergence), and line AB to the boundary of dynamic loss of stability (due to flutter).

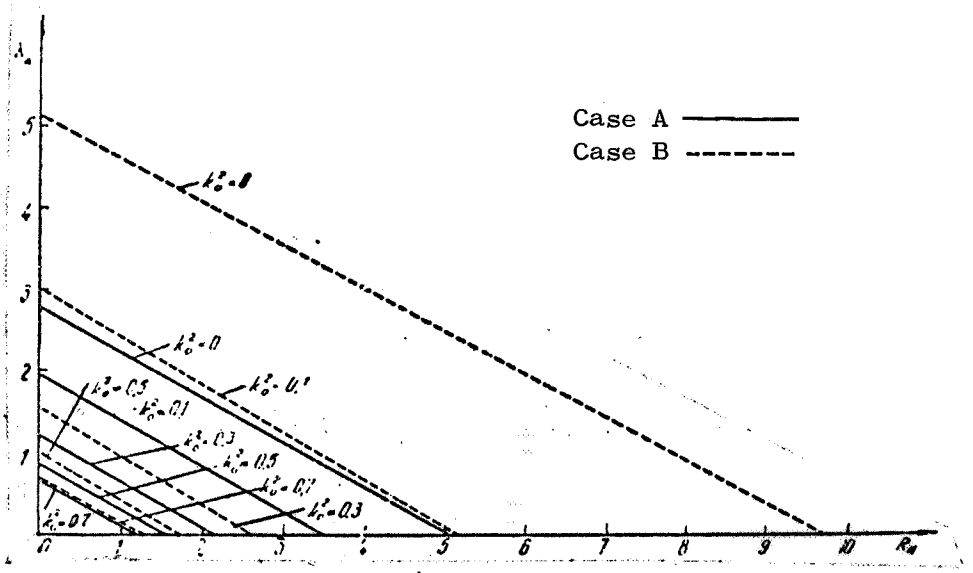

Figure 4.

The relation $w^2 \sim \lambda$ for various values of k_0^2 and R_{11} is shown in Figure 6a-d, relations (39) and (43) being used.

Lastly, Figure 7a-f shows, on the basis of relation (38), the relation $h_* \sim M_\infty$ for various values of the Alfven number and parameters k_0^2 and R_{11}.

6. Conclusions and Discussion

An analysis has been presented of the problem of aeroelastic stability of symmetrically built plane structures of the sandwich type placed in a current of ideally conducting supersonic gas.

It has been shown that in the context of the present problem the aerodynamic terms contain, in addition to the term $\mathcal{A}(w)$ (which in the case of modal representation of the solution implies coupling of the modes only in direction ξ_1), also the term $k^2 \Delta \mathcal{A}(w)$, which in the same circumstances implies coupling of the modes in direction ξ_2 as well. It is to be seen that if, when $k_0^2 = 0$ or $a/b = 0$, there occurs only coupling of the modes in direction ξ_1 with increase

in k_0^2 or a/b, coupling of the modes in direction ξ_2 will become increasingly stronger. This leads to direct justification of the conclusion drawn by Anderson [6] and Erickson and Anderson [7], to the effect that use of the Galerkin method in study of the present problem, a solution of two terms being considered, leads to errors which increase in comparison to the exact solution along with increase in k_0^2 or a/b.

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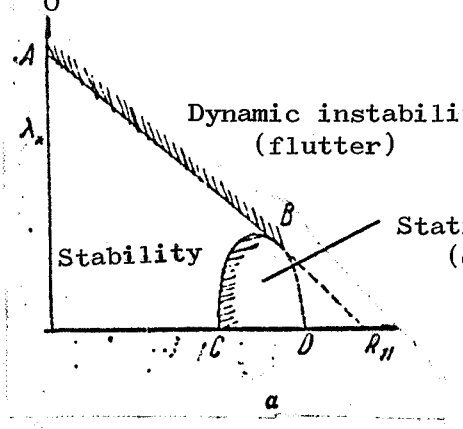


a

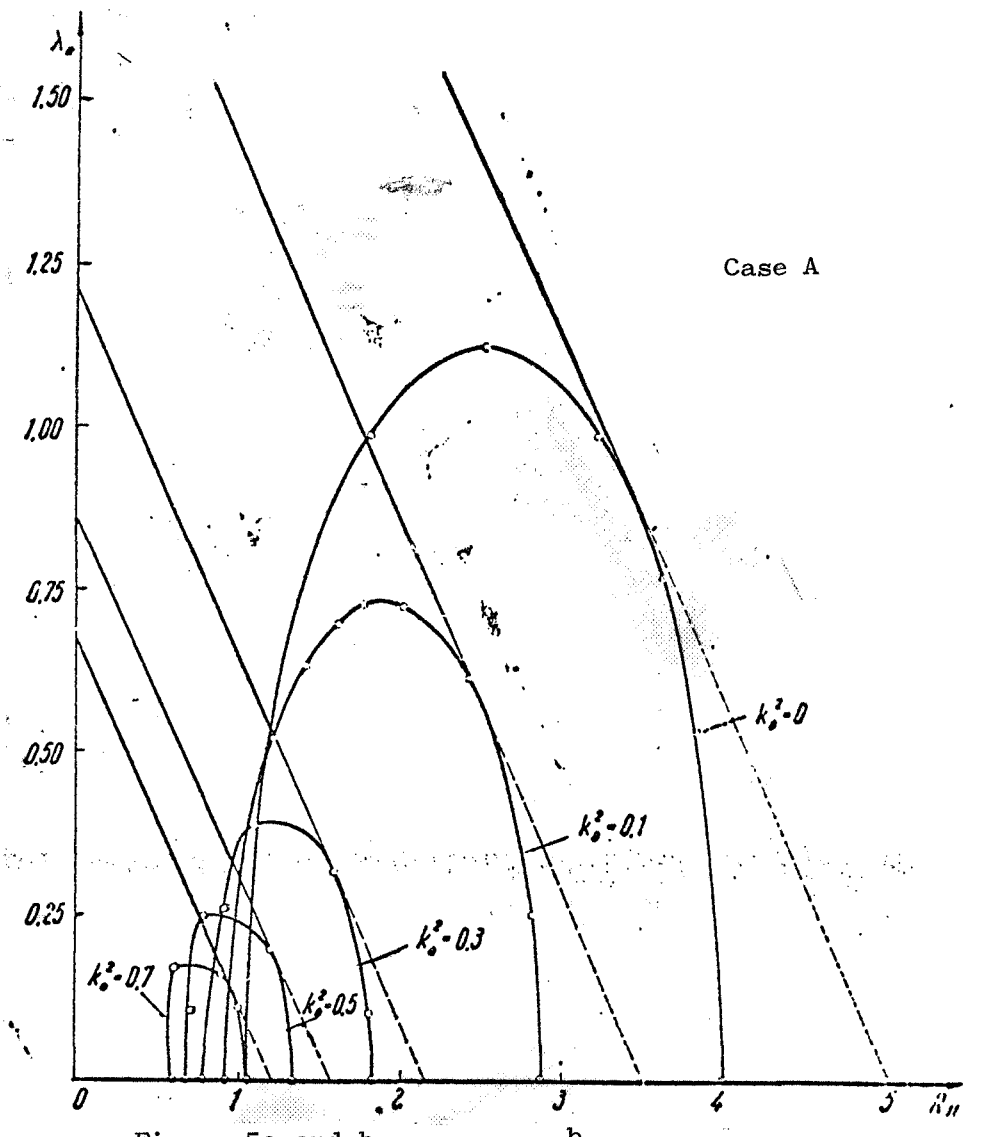
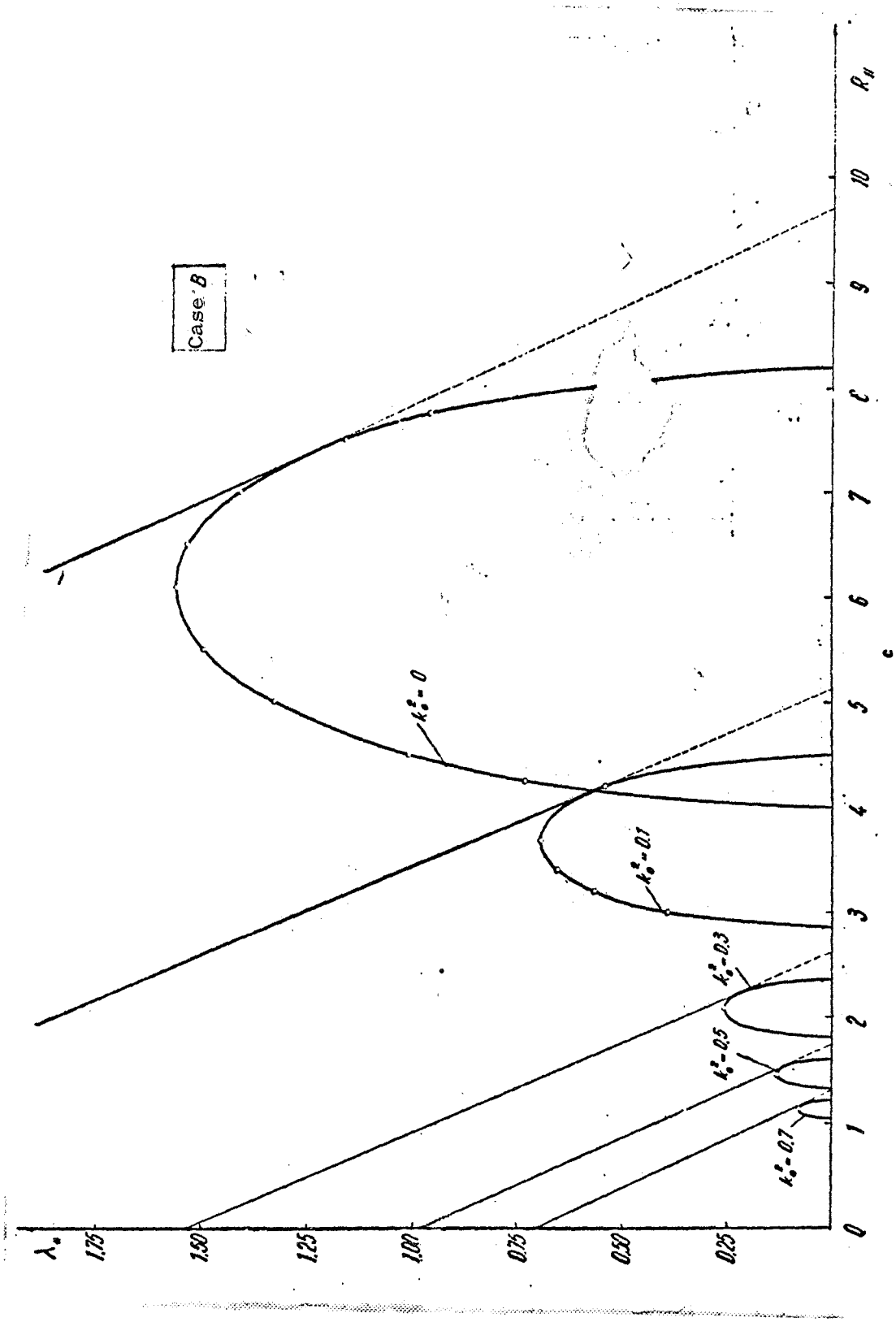


Figure 5a and b

b


Case B
λ_*
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0
1
2
3
4
5
6
7
8
9
10
R_H
$k_0^2 = 0$
$k_0^2 = 0.1$
$k_0^2 = 0.3$
$k_0^2 = 0.5$
$k_0^2 = 0.7$

c
Fig. 5c

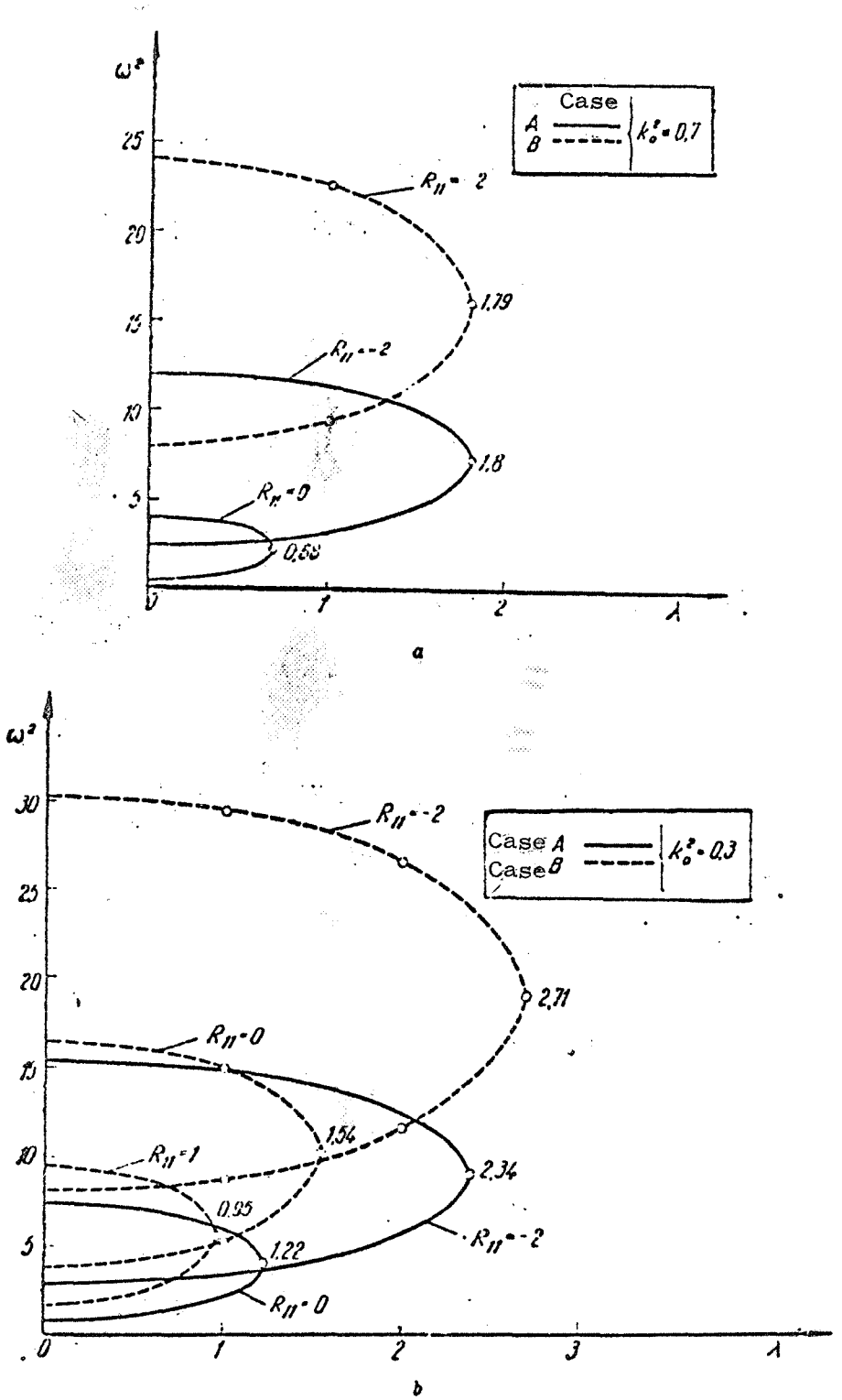


Figure 6a and b

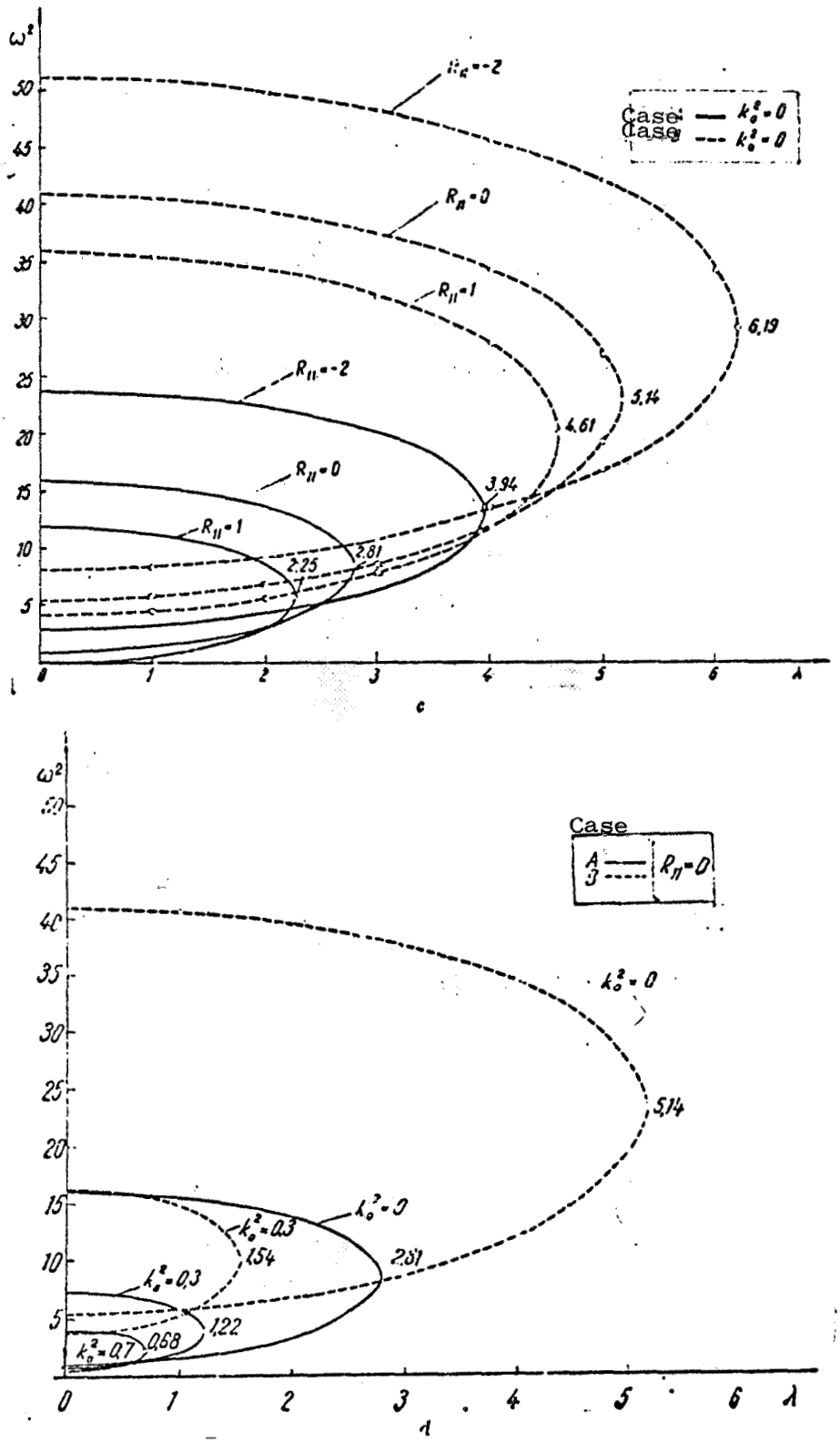


Figure 6c and d

In order to avoid possible confusion, we shall replace the formulation of /190
the conclusion drawn by Anderson [6] and Erickson and Anderson [7] (it appears to be obvious in this connection that consideration of two terms in development of (9) is perfectly justifiable when $a/b = 0$) with the statement that when the Galerkin method is used in study of this problem, function $w(\xi_1, \xi_2, t)$ should be represented by relation (9) and not by (9').

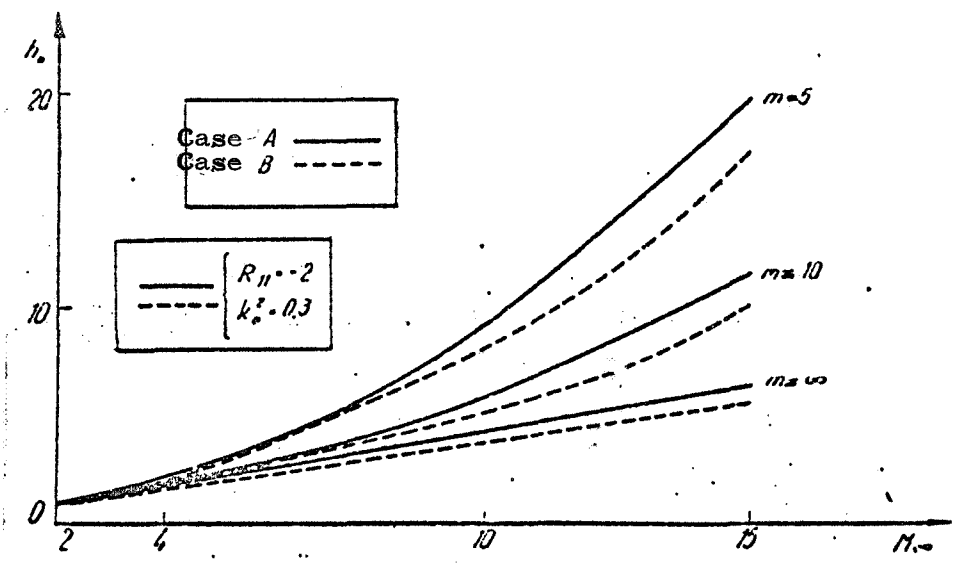
In order to avoid particularly laborious developments in this study, the analysis has been rendered special for the case of an infinite band ($a/b = 0$). We may mention that such specialization of the problem does not limit the value of the conclusions obtained; from the qualitative standpoint they remain valid for the problem of rectangular plane panels as well.

Figures 3 and 4 point up the fact that the flutter boundary is highly sensitive to variation in the parameter k_0^2 in the range $0 < k_0^2 < 0.7$ (this conclusion also follows from the studies undertaken in [3], [4], [9]).

It also follows that in the case of a compressed infinite sandwich band ($R_{11} > 0$), a decrease in λ_{*f1} corresponds to increase in k_0^2.

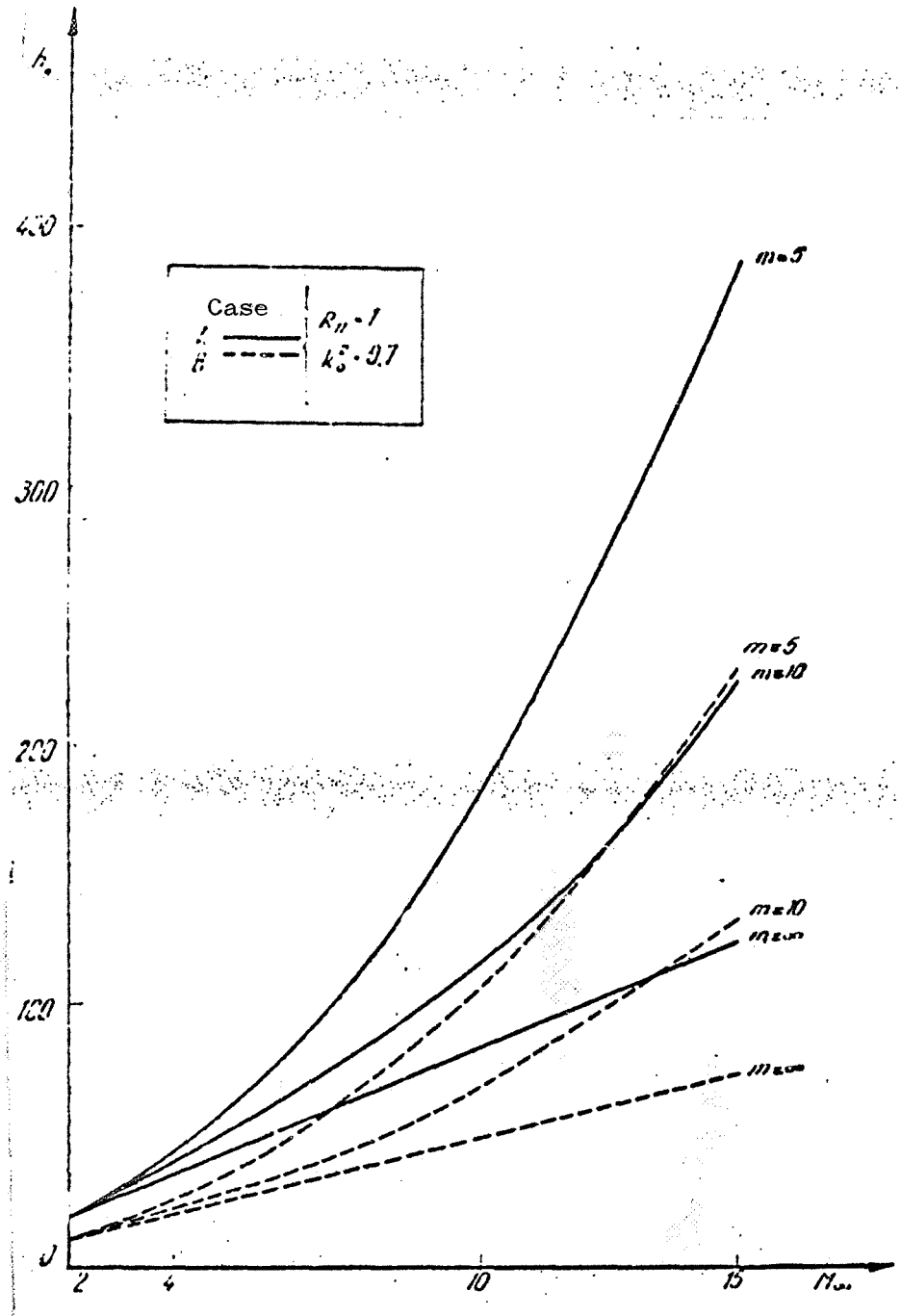
The same result is also valid in the case corresponding to $R_{11} = 0$. For the case of a sandwich band under tension ($R_{11} < 0$), there correspond to the parameters $k_0^2 > 0$ flutter velocity values which are larger than in the case of a band of identical geometry but rigid in shear ($k_0^2 = 0$). It is to be observed that the anteflutter region is much larger in case B (the case corresponding to a band rigidly fixed at the edges, $\xi_1 = 0, a$) than in case A (a band having simply supported edges, $\xi_1 = 0, a$).

This characteristic, strongly pronounced in the case of a band rigid in shear, is attenuated with increase in k_0^2, so that around the value $k_0^2 = 0.7$ the anteflutter region obtained in case A virtually coincides with that of case B.



a

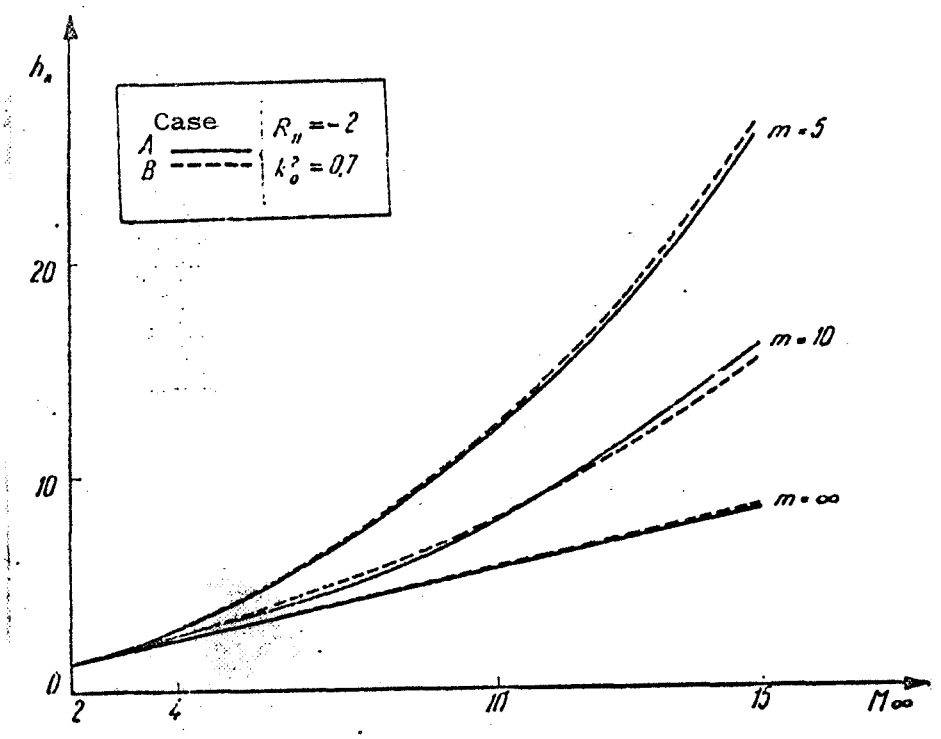
Fig. 7 a


h_*
400
300
200
100
0
Case
A
B
R_{II} - 1
k_3^5 - 9.7
m=5
m=5
m=10
m=10
2
4
10
15
M_∞

b

Fig. 7 b

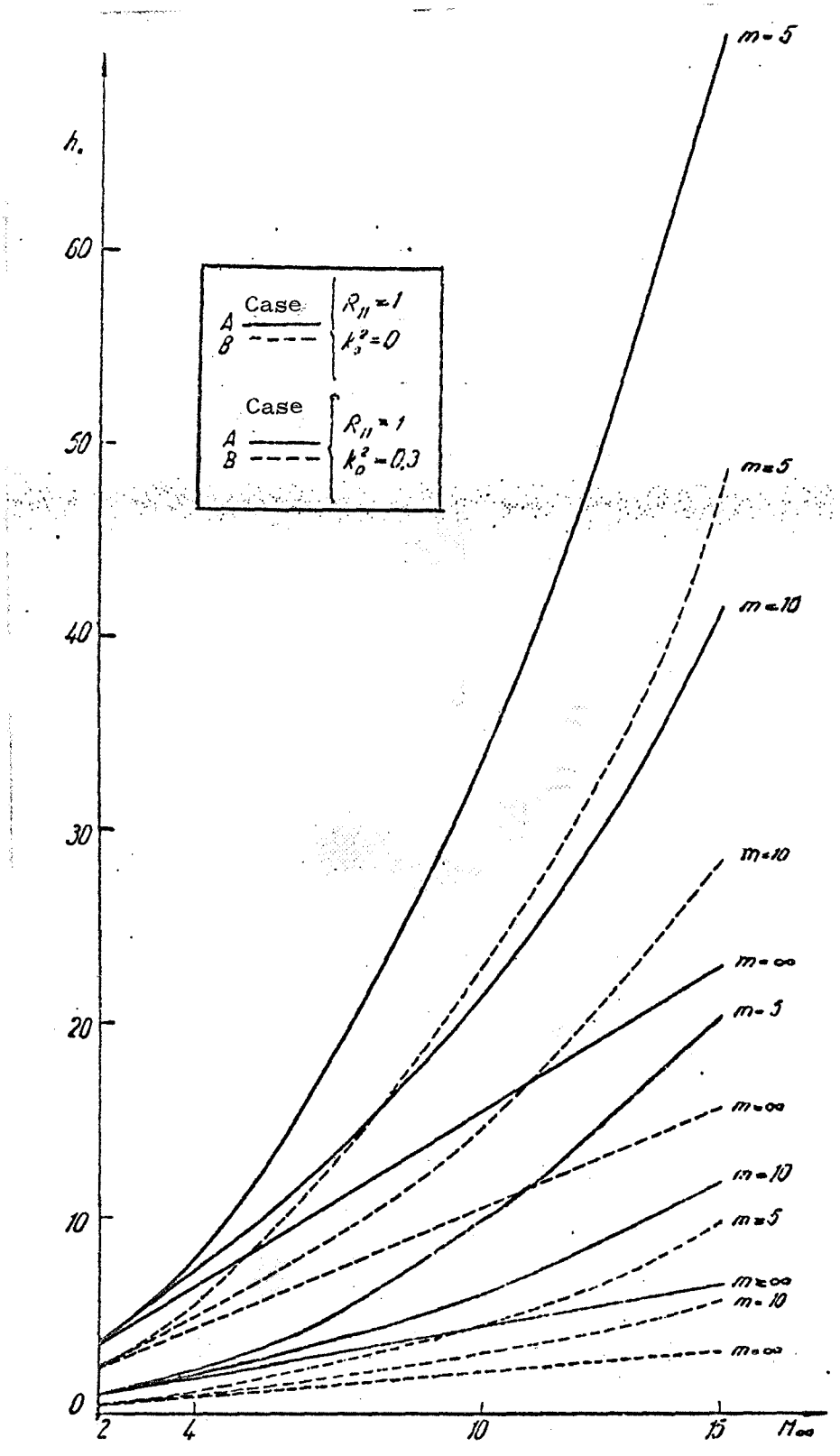
Figures 6a-d also clarify the influence of various parameters on the flutter /192 behavior of an infinite sandwich band. It is found that for a given value R_{11} decrease in transverse shear rigidity (increase in k_0^2) causes decrease in the critical flutter velocity and frequency, as well as in the natural vibrational frequencies of the sandwich band. The influence exerted by the support conditions considered (identified by us as cases A and B) on the flutter behavior of an infinite sandwich band is also brought to light. It is observed that for a given k_0^2 and R_{11} the flutter frequency corresponding to case A is much lower than that corresponding to case B, a property maintained throughout the k_0^2 value range considered. Comparison of the results obtained for the two support cases shows that in the case of structures but little flexible in shear the flutter velocity in case B is much lower than that corresponding to case A, the property tending to diminish along with increase in k_0^2.



c

Fig. 7 c

Figures 5a and b additionally contribute toward illustration of the influence of parameter k_0^2 in the problem of aeroelastic stability of a compressed sandwich band in the two support cases A and B. The presence of a current of gas is found to exert a stabilizing effect as regards loss of stability due to divergence (this being pointed up by the circumstance that in a certain velocity zone the critical compressive tension R_{11} of a panel placed in a current of gas is higher than in the absence of a current of gas).


h.
60
50
40
30
20
10
0
2
4
10
15
M∞
Case
A
B
R11=1
k3²=0
Case
A
B
R11=1
k0²=0.3
m=5
m=5
m=10
m=10
m=∞
m=5
m=∞
m=10
m=5
m=∞
m=10
m=∞

d

Figure 7 d

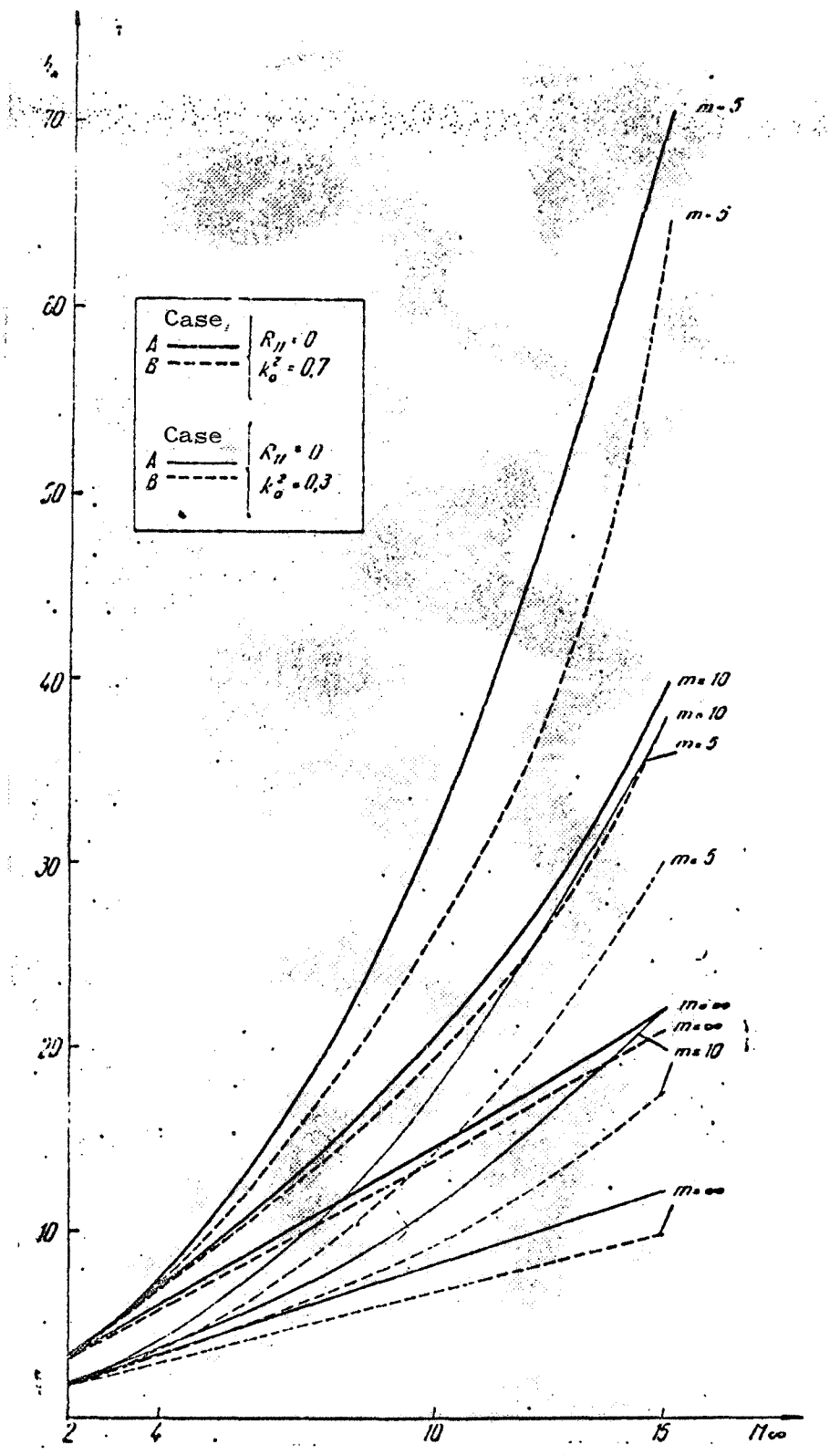
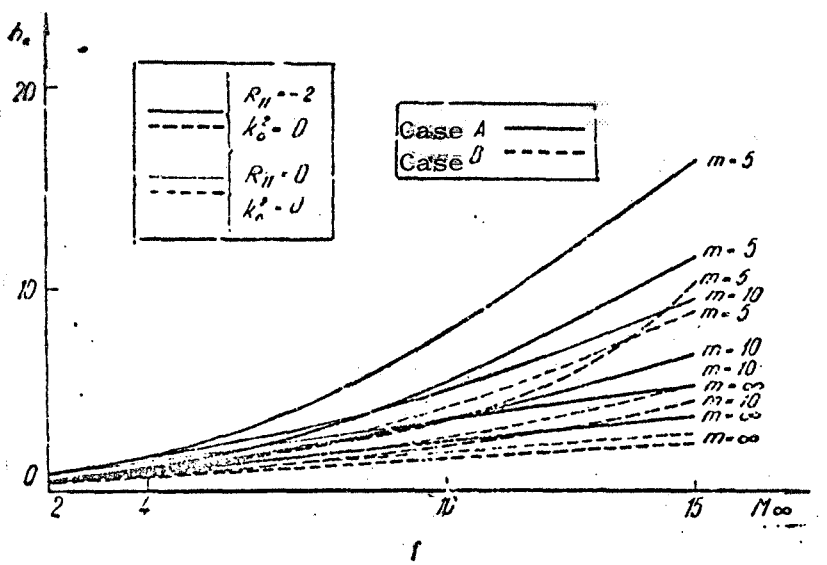

70
60
50
40
30
20
10
Case I
A
B
$R_{II} = 0$
$k_0^2 = 0.7$
Case II
A
B
$R_{II} = 0$
$k_0^2 = 0.3$
m = 5
m = 5
m = 10
m = 10
m = 5
m = 5
m = ∞
m = ∞
m = 10
m = ∞
2
4
10
15
M∞
c

Figure 7e

This property holds for all the values of k_0^2 and in all the support cases considered. The property nevertheless appears to be more pronounced in the case of a band but little flexible in transverse shear. /195

Figures 7a-f lead again to the conclusion formulated by Lisunov [16] and Efremov [9]; namely that a decrease in the critical flutter velocity corresponds to increase in the magnetic field (decrease in the Alfven number), a property which holds for all the values of parameters k_0^2 and R_{11} considered. Corresponding to a given Alfven number m and parameter R_{11}, along with decrease in k_0^2 there is an increase in the critical flutter velocity, one which is more pronounced in support case B than in case A.

Figure 7c shows the range of variation of parameters R_{11} and k_0^2, wherein the flutter velocity of case A coincides with that of case B for given values of the Alfven number.



f

Fig. 7 f

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